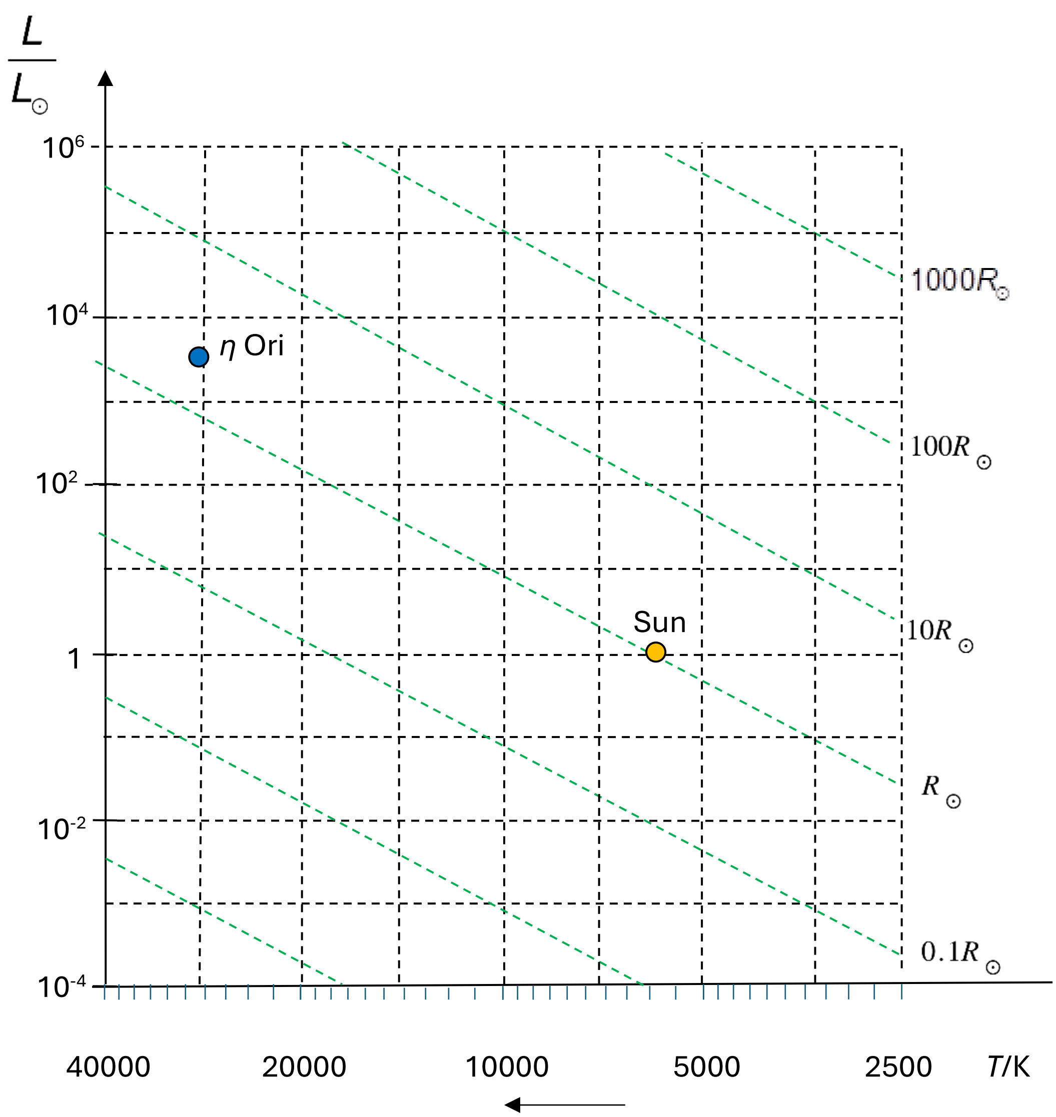
An HR diagram worksheet

The Sun and the star η Ori are shown on this incomplete HR diagram. For the Sun, $L_{\odot} = 3.8 \times 10^{26}$ W .



(a) Determine for η Ori

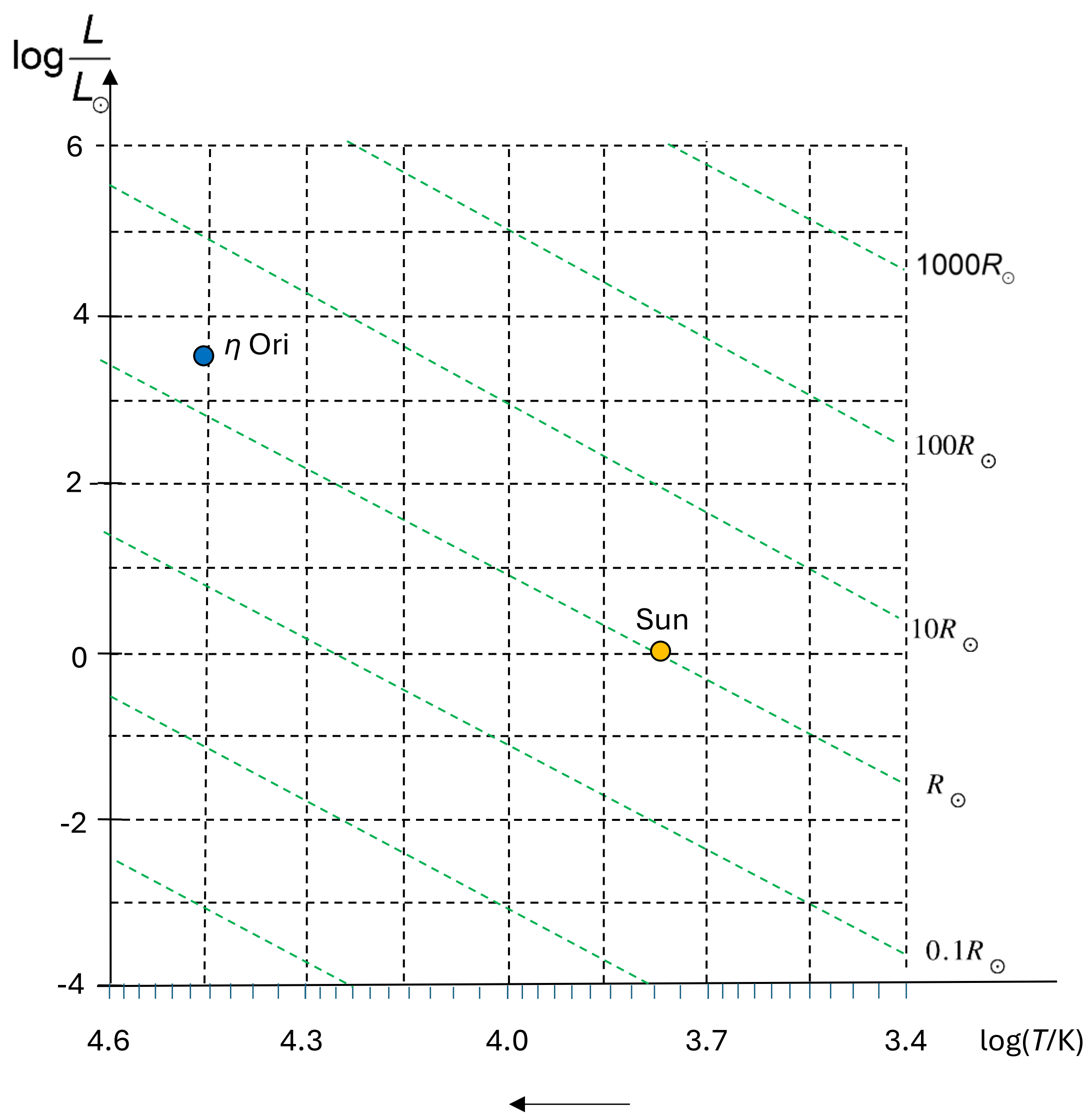
(i) the luminosity in W,
(ii) the temperature in K,
(iii) the radius in m.

(b) The peak wavelength in η Ori's spectrum.

Answers

(a) We must understand that the HR diagram is a plot of $\log\frac{L}{L_\odot}$ versus $\log(\frac{T}{K})$.

The diagram then looks like the one below. The numbers on these axes are the logarithms of the numbers in the original diagram base 10:



(i) The vertical coordinate of the star is half-way between $\log 10^4 = 4$ and $\log 10^3 = 3$, i.e. it is 3.5. This means that $\log \frac{L}{L_\odot} = 3.5$ and so $\frac{L}{L_\odot} = 10^{3.5} = 3.16 \times 10^3$. Finally,

$L = 3.16 \times 10^3 \times L_\odot = 3.16 \times 10^3 \times 3.8 \times 10^{26} = 1.2 \times 10^{30}$ W.

(ii) The horizontal coordinate of the star is half-way between $\log 40000 = 4.60$ and $\log 20000 = 4.30$ i.e. 4.45. Hence $\log(\frac{T}{\text{K}}) = 4.45$ and so $\frac{T}{\text{K}} = 10^{4.45} \Rightarrow T = 2.8 \times 10^4$ K.

Equivalently, the horizontal axis has ticks.

(The ticks are not equally separated yet they represent the same change in temperature. This is what is meant by a logarithmic graph. The change in temperature from tick to next tick is 250 K for the interval 2500 K to 5000 K, 500 K for the interval 5000 K to 10000 K, 1000 K for the interval 10000 to 20000 K and 2000 K for the interval 20000 K to 40000 K. It is highly unlikely that an exam HR diagram will have these ticks.)

We see that the star falls just to the left of the fourth tick after 20000 K, so its temperature is just above 20000 + 4×2000 = 28000 K.

(iii) Since $L = \sigma 4\pi R^2 T^4$ it follows that

$R = \sqrt{\frac{L}{\sigma 4\pi T^4}} = \sqrt{\frac{1.2 \times 10^{30}}{5.67 \times 10^{-8} \times 4\pi \times (2.8 \times 10^4)^4}} = 1.7 \times 10^9$ m. (This means that $\frac{R}{R_\odot} = \frac{1.7 \times 10^9}{7.0 \times 10^8} = 2.4$, consistent with the HR diagram.)

(b) From Wien's law, $\lambda = \frac{2.9 \times 10^{-3}}{2.8 \times 10^4} \approx 1.0 \times 10^{-7}$ m, a UV wavelength.